

# Wasserstein-based fairness interpretability framework for machine learning models

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North Carolina State University, Mathematics Department, Numerical Analysis Seminar, March 2022

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# Overview

- Introduction
- Classifier fairness
- Regressor fairness
- ML interpretability
- Fairness interpretability
- Bias mitigation with regulatory constraints

# Introduction

- Predictive ML models, and strategies that rely on such models, are subject to laws and regulations that ensure fairness (e.g. ECOA, EEOA).
- Examples of protected attributes: [race](#), [gender](#), [age](#), [ethnicity](#), [national origin](#), [marital status](#), etc.
- Tradeoff between accuracy and bias.

## Main steps in ML fairness

1. Fairness assessment (or bias measurement).
2. Bias mitigation.

# Setup

## Data $(X, G, Y)$

- $X \in \mathbb{R}^n$ , predictors
- $G \in \{0,1\}$  (e.g. male/female)
- $Y \in \{0,1\}$  or  $Y \in \mathbb{R}$ , response variable

## Models

- $f(X) = \hat{\mathbb{P}}(Y = 1|X)$  or  $\hat{\mathbb{E}}(Y|X)$  trained regressor
- $Y_t = 1_{\{f(X) > t\}}$ , a classifier for a given threshold  $t \in \mathbb{R}$
- $\hat{Y}$ , a classifier

## Labels

- Non-protected class:  $G = 0$
- Favorable outcome:  $Y = 0$

# Classifier fairness

- ML bias can be viewed as an ability to differentiate between subpopulations at the level of data or outcomes (*Dwork et al 2012*)

- Statistical parity (*Feldman et al, 2015*)

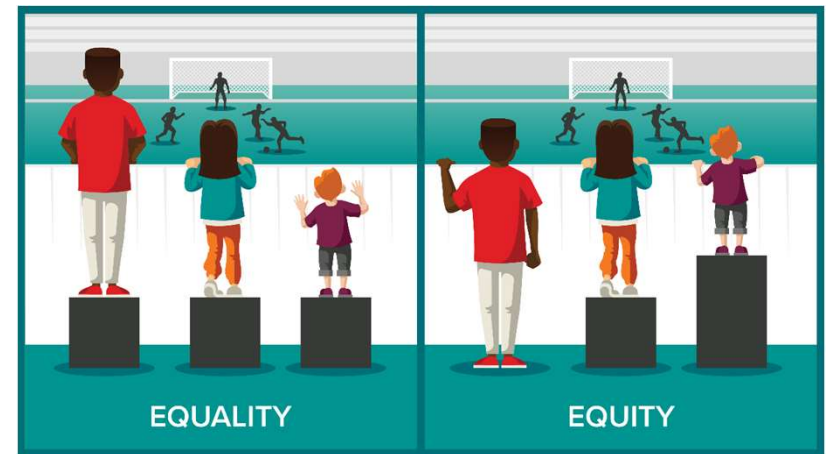
$$\mathbb{P}(\hat{Y} = 0|G = 0) = \mathbb{P}(\hat{Y} = 0|G = 1)$$

- Equalized odds (*Hardt et al, 2015*)

$$\mathbb{P}(\hat{Y} = 0|Y = y, G = 0) = \mathbb{P}(\hat{Y} = 0|Y = y, G = 1), y \in \{0,1\}$$

- Equal opportunity (*Hardt et al, 2015*)

$$\mathbb{P}(\hat{Y} = 0|Y = 0, G = 0) = \mathbb{P}(\hat{Y} = 0|Y = 0, G = 1)$$



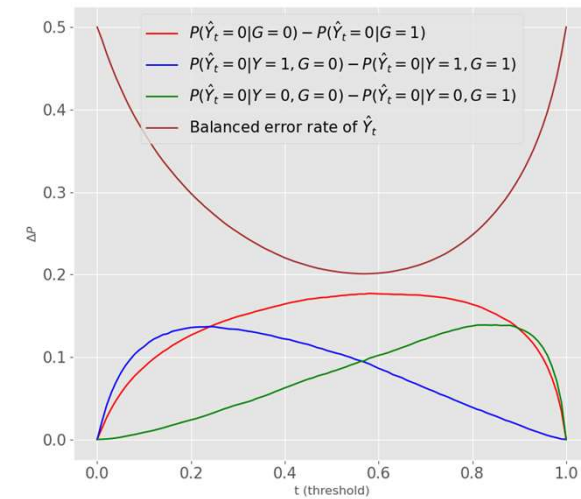
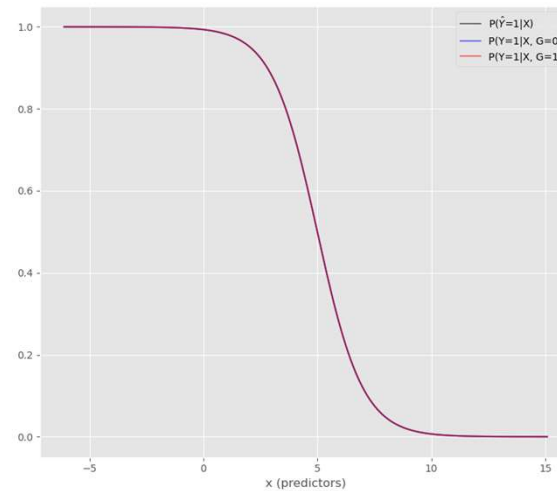
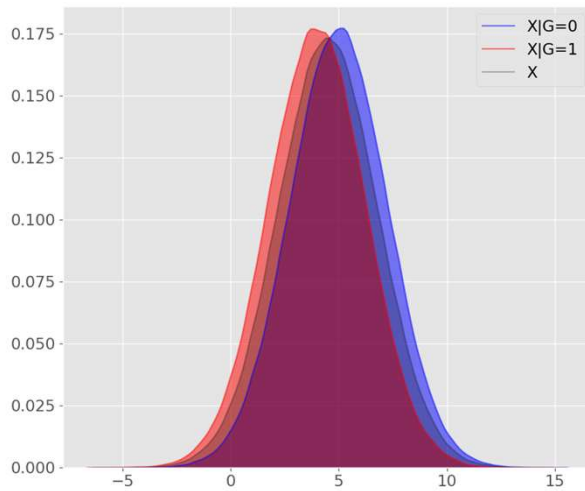
# Classifiers fairness

## Statistical parity classifier bias

$$\text{bias}(Y_t|X, G) = |\mathbb{P}(Y_t = 0|G = 0) - \mathbb{P}(Y_t = 0|G = 1)|$$

### Example (proxy predictor)

- $X \sim N(5 - G, \sqrt{5})$  ,  $\mathbb{P}(G = 0) = \mathbb{P}(G = 1) = 0.5$
- $Y \sim \text{Bernoulli}(f(X)), f(x) = \text{logistic}(5 - x)$



## Fairness with awareness

Selected approaches for bias reduction in classifiers with access to protected attributes

- Maximization with fairness constraints

$$Y^*(X, G) \text{ or } Y^*(X) = \max_{\text{fairness}(Y^*|G)} \mathbb{E}[\mathcal{L}(Y^*, X^{(train)})]$$

Dwork et al (2012), Woodworth et al (2017), Zhang et al (2018), and many others.

- Post-corrective methods (Hardt et al, 2015)
  - Study of equalized odds, equal opportunity, statistical parity
  - Construction of fair randomized classifier  $\tilde{Y}(X, G; f) \in \mathcal{P}(\{0,1\})$  via post-processing
- Dataset repairment via optimal transport. Feldman et al (2015), Gordaliza et al. (2019).

## Fairness with awareness

Fair dataset construction. [Feldman et al \(2015\)](#)

- Geometric repair:  $X_i|G = k$  moving towards Wasserstein barycenter  $\tilde{X}_i$ .
- Training a classifier on repaired dataset  $\tilde{X}(X, G, \lambda)$ ,  $\lambda \in [0,1]$
- Fairer predictors imply fairer classifier
- Useful when  $Y$  is not available

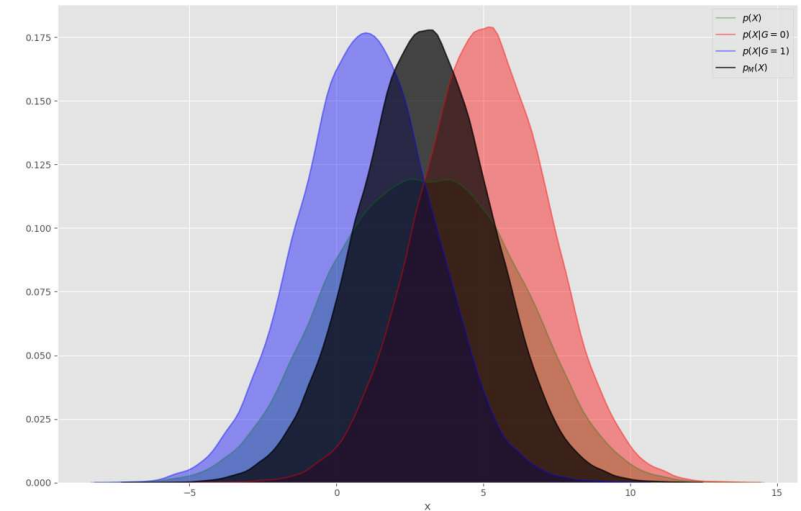
Random repair. [Gordaliza et al \(2019\)](#)

- Controlling the statistical parity bias via geometric repair is difficult
- Control must be via TV-distance:

$$\text{bias}^C(\hat{Y}|X, G) \leq d_{TV}(P_{X|G=0}, P_{X|G=1})$$

- Random repair picks at random, *Bernoulli*( $\lambda$ ), between samples of  $P_{X|G=k}$  and the barycenter of subpopulations:

$$\text{bias}^C(\hat{Y}|\tilde{X}_\lambda, G) \leq d_{TV}(P_{\tilde{X}|G=0}, P_{\tilde{X}|G=1}) = 1 - \lambda$$





# Motivation

## Comments

- Bias measurements test fairness of predictors  $X$  or a classifier  $\hat{Y}$ , not the regressor  $f(X)$
- Mitigation procedures focus on the construction of fair classifiers  $\hat{Y}^*(X, G)$ , not a fair regressor.

## Regulatory constraints. Fairness without awareness.

- $G$  is typically not collected.
- Training with access to  $G$  is not allowed.
- Models (including post-processed ones)  $f(X, G)$  that require access to  $G$  are not allowed.

## Proxy models of $G$ for validation

- Certain proxy models  $\tilde{G}$  for  $G$  are allowed for validation by compliance office.  $\tilde{G}$  is prohibited to share outside of it.
- Postprocessing is possible by compliance but the model  $\tilde{f}(X)$  must rely on  $X$  only. No leakage of  $(X, \tilde{G})$  is allowed.

## Objectives of our work

Given a trained model regressor or classification score  $f(X)$ :

1. **Measurement.** Evaluate regressor bias.
2. **Bias Interpretability.** Quantify the contribution of each predictor to that bias.
3. **Mitigation.** Produce family of post-processed models  $\{\tilde{f}_\alpha(X; f)\}$  using a proxy model  $\tilde{G}$ .

## Regressor bias

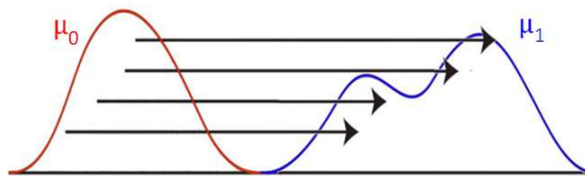
### Model (regressor) bias

$$\text{Bias}_{W_1}(f|X, G) = W_1(f(X)|G = 0, f(X)|G = 1)$$

- Wasserstein metric  $W_1$  (optimal transport cost)

$$W_1(\mu_0, \mu_1) = \inf_{\pi \in \mathcal{P}(\mathcal{Z}^2)} \left\{ \int |z_1 - z_2| \pi(dz_1, dz_2), \pi \text{ with marginals } \mu_0, \mu_1 \right\}$$

- $\mu_k = P_{f(X)|G=k}, k \in \{0,1\}$



**Note:** The main focus is on the bias in the output (model), not the input (predictors).

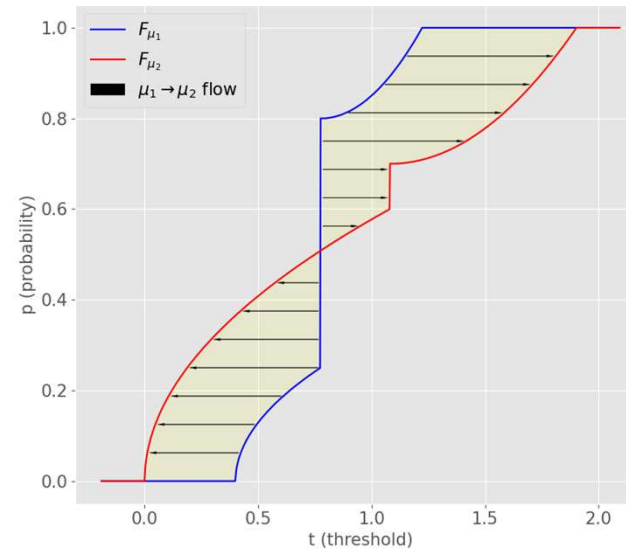
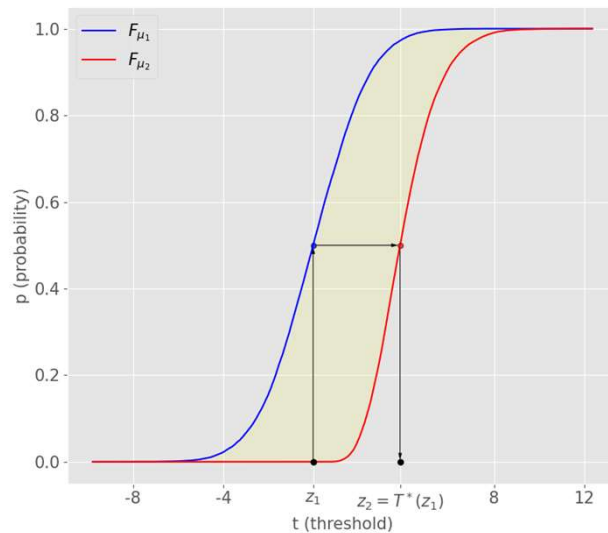
## Model bias metrics

### Basic properties (one-dimension)

- $\mu_1, \mu_2$  on  $\mathcal{B}(\mathbb{R})$ , there exists order preserving optimal transport plan  $\pi^*$  such that

$$W_1(\mu_1, \mu_2) = \int |x_1 - x_2| d\pi^* = \int \left| F_{\mu_1}^{[-1]}(p) - F_{\mu_2}^{[-1]}(p) \right| dp = \int |F_{\mu_1}(t) - F_{\mu_2}(t)| dt$$

- Transport map vs transport plan:



## Positive and negative flows

Need to understand whether the model favors majority class or minority one.

Assumption: Model  $f(X) \in \mathbb{R}$  has a favorable direction  $\zeta_f = \pm 1$ .

Definition:  $Bias_{W_1}^{\pm}(f|X, G)$  is the cost of transporting  $P_{f(X)|G=0}$  in favorable/non-favorable directions.

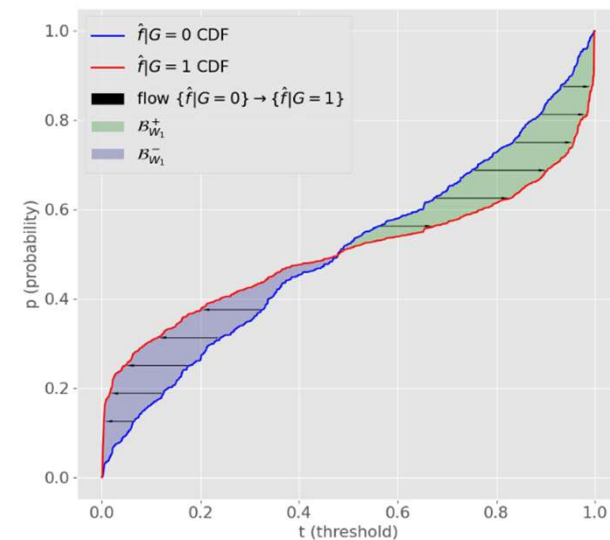
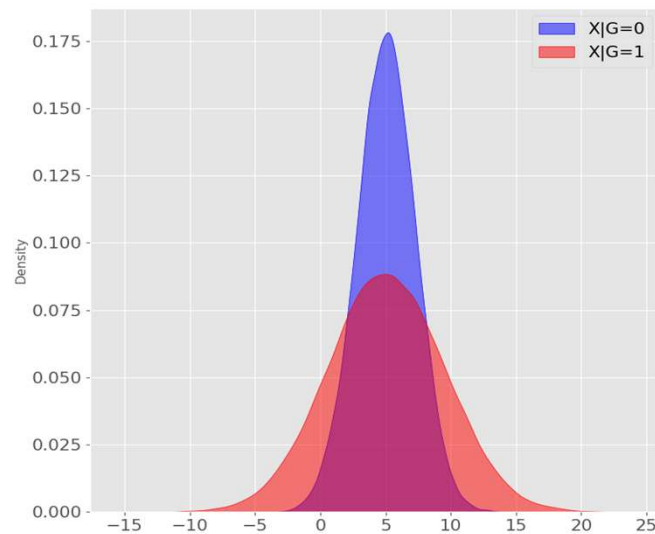
Example:

$$X \sim \mathcal{N}(\mu, (1 + G)\sqrt{\mu})$$

$$Y \sim \text{Bernoulli}(f(X))$$

$$f(X) = \sigma(\mu - X)$$

$$\zeta_f = -1$$



## Model bias metrics

Facts [Miroshnikov et al, 2021a]

- Integrated statistical parity bias:

- $Bias_{W_1}(f|X, G) = \int bias(Y_t|X, G)dt$

- $Bias_{W_1}^{\pm}(f|X, G) = \int_{\mathcal{T}_{\pm}} bias(Y_t|X, G) dt$

- Integrated generic parity bias:  $\mathcal{A} = \{A_1, \dots, A_M\}$ ,  $\mathbb{P}(Y_t = 1|G = 0, A_m) = \mathbb{P}(Y_t = 1|G = 1, A_m)$ ,  $A_m \in \mathcal{A}$

$$Bias_{W_1, \mathcal{A}}(f|X, G) = \sum w_m W_1(f(X)|\{G = 0, A_m\}, f(X)|\{G = 1, A_m\}) = \int bias_{\mathcal{A}}(Y_t|X, G)dt$$

## Input-output bias relationship

- Bias in predictors propagates through the model:

$$\text{Bias}_{W_1}(f|X, G) \leq [f]_{Lip} W_1(X|G=0, X|G=1)$$

- Fairness of predictors is sufficient for model fairness, but not necessary:

$$X_1 \sim N(\sqrt{\tau} \cdot G, 1), \quad X_2 \sim N(1, 1), \quad Y = \frac{1}{\tau} X_1 + X_2$$

Here  $\text{Bias}(Y|X, G) \rightarrow 0$  and  $\text{Bias}(X|G) \rightarrow \infty$  as  $\tau \rightarrow \infty$ .

- We would like to understand how each predictor contributes to the model bias  $\text{Bias}_{W_1}(f|X, G)$ .

## Model explanations

To design fairness interpretability we first review model explanations.

### Basic post-hoc model explainers.

Given  $f$  and  $X \in \mathbb{R}^n$ , the contribution of  $X_i$  to  $f(X)$  can be quantified by

- $E_i^{ME}(X; f) = \mathbb{E}[f(x_i, X_{-\{i\}})]|_{x_i=X_i}$ , marginal expectation (ME), [PDP, Freidman, 2001]
- $E_i^{CE}(X; f) = \mathbb{E}[f(X)|X_i]$ , conditional expectation (CE)

**Note:** Marginal explains  $x \rightarrow f(x)$  and the conditional  $X(\omega) \rightarrow f(X(\omega))$ .



## Model explanations

### Post-hoc explainers (game-theoretical)

- Players:  $N = \{1, 2, \dots, n\}$  (features become player)
- Game: set function  $v(S)$ ,  $S \subset N$ ,  $v(N)$  = total payoff
- Shapley value [Shapley, 1953]

$$\varphi_i[v] = \sum_{S \subset N} \frac{(s-1)!(n-s)!}{n!} (v(S) - v(S \setminus \{i\})), \quad i \in N$$

$\varphi$  is efficient:  $\sum_i \varphi_i[v] = v(N)$ , linear, symmetric.

### Probabilistic games

- $v^{CE}(S; X, f) = \mathbb{E}[f(X_S, X_{-S}) | X_S]$ , conditional game explores model predictions
- $v^{ME}(S; X, f) = \mathbb{E}[f(x_S, X_{-S})] |_{x_S = X_S}$ , marginal game explores the model

## Fairness Interpretability

### Definition (basic bias explanations)

- Given an explainer  $E_i(X; f)$  of predictor  $X_i$ , the bias explanation is defined via the transport cost

$$\beta_i(f|X, G) = W_1(E_i(X)|G = 0, E_i(X)|G = 1)$$

- Positive and negative bias explanations  $\beta_i^\pm$  are defined as transport effort in favorable and non-favorable

directions: 
$$\beta_i^\pm = \int_{\mathcal{P}_{i^\pm}} \left| F_{E_i|G=0}^{[-1]}(p) - F_{E_i|G=1}^{[-1]}(p) \right| dp$$

### Notes

- Type of ML explainers matters (marginal vs conditional)
- $\beta_+$  quantifies the positive contribution (increase in positive flow and decrease in negative)

# Fairness Interpretability

Example: basic bias explanations based on marginal Shapley model explainer

$$\mu = 5, a = \frac{1}{20} (10, -4, 16, 1, -3)$$

$$X_1 \sim \mathcal{N}(\mu - a_1(1 - G), 0.5 + G)$$

$$X_2 \sim \mathcal{N}(\mu - a_2(1 - G), 1)$$

$$X_3 \sim \mathcal{N}(\mu - a_3(1 - G), 1)$$

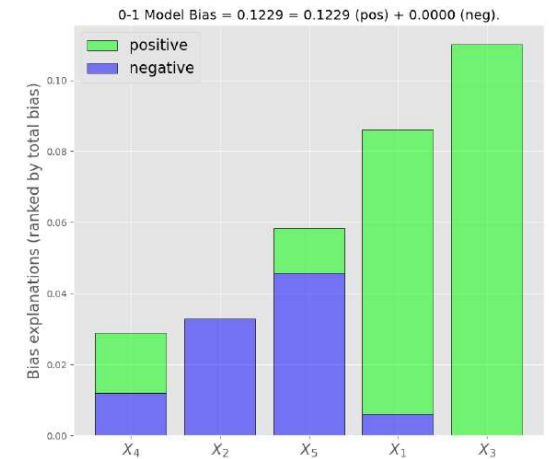
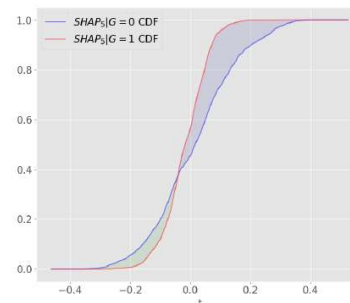
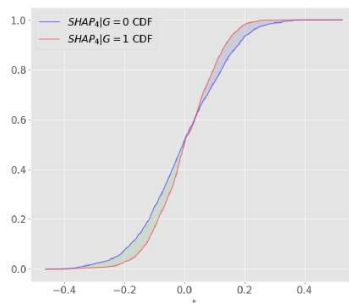
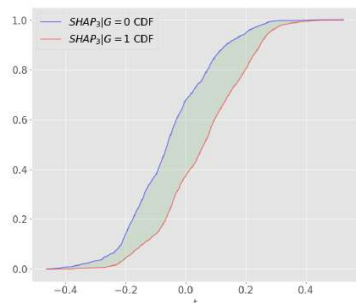
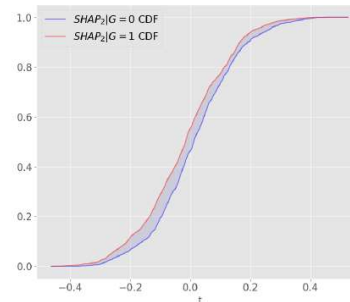
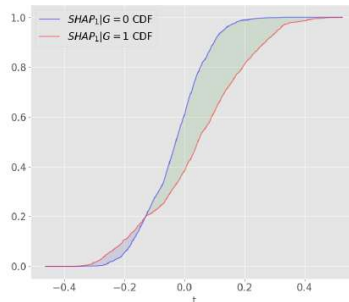
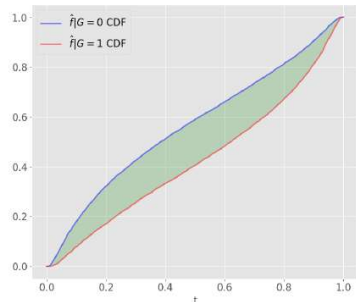
$$X_4 \sim \mathcal{N}(\mu - a_4(1 - G), 1 - 0.5G)$$

$$X_5 \sim \mathcal{N}(\mu - a_5(1 - G), 1 - 0.75G)$$

$$Y \sim \text{Bernoulli}(f(X))$$

$$f(X) = \sigma(\sum X_i - 24.5)$$

$$\zeta_f = -1$$



## Fairness Interpretability

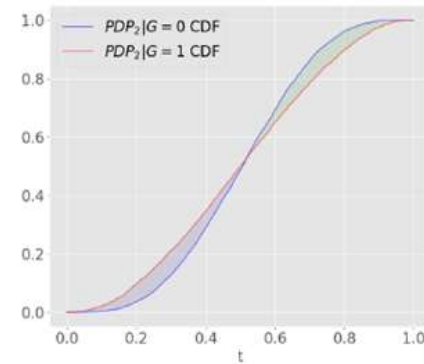
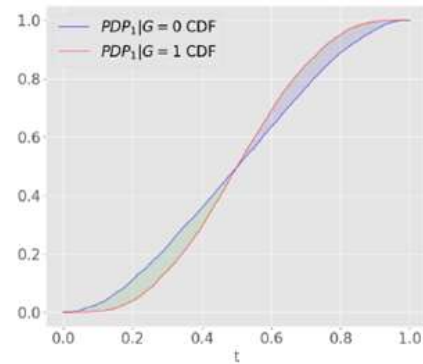
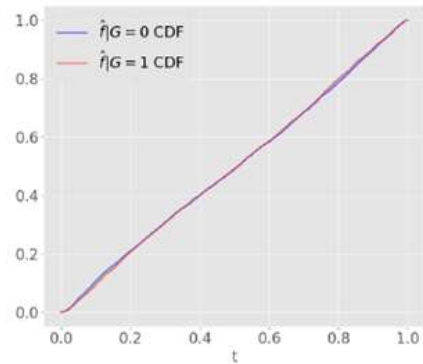
### Example (bias offsetting)

$$X_1 \sim \mathcal{N}(\mu, 2 - G)$$

$$X_2 \sim \mathcal{N}(\mu, 1 + G)$$

$$Y \sim \text{Bernoulli}(f(X))$$

$$f(X) = \sigma(2\mu - X_1 - X_2)$$



## Fairness Interpretability

- Basic bias explanations are not additive.
- Do not explain the direct contribution to the negative and positive model bias.

### Game theoretical approach

- Consider an ML explainer  $E_S(X; f)$  of predictor  $X_S$ ,  $S \subset \{1, 2, \dots, n\}$
- Predictors  $\{X_i\}_{i \in N}$  are players that push/pull explainer subpopulation distributions apart when joining a coalition  $S \subset N$
- A game  $v^{bias}(S) = Bias_{W_1}(E_S(X)|G) = W_1(E_S(X)|G = 0, E_S(X)|G = 1)$
- A game  $v^{bias\pm}(S) = Bias_{W_1}^{\pm}(E_S(X)|G)$
- Shapley bias explanations  $\varphi^{bias}(f|X, G) = \varphi[v^{bias}]$ ,  $\varphi^{bias\pm}(f|X, G) = \varphi[v^{bias\pm}]$

$$Bias_{W_1}^{\pm}(f|X, G) = \sum_i \varphi^{bias\pm}(f|X, G)$$

**Note:** explanations are signed and additive

# Fairness Interpretability

## Example (marginal Shapley-bias explanations)

$$\mu = 5, a = \frac{1}{20}(10, -4, 16, 1, -3)$$

$$X_1 \sim \mathcal{N}(\mu - a_1(1 - G), 0.5 + G)$$

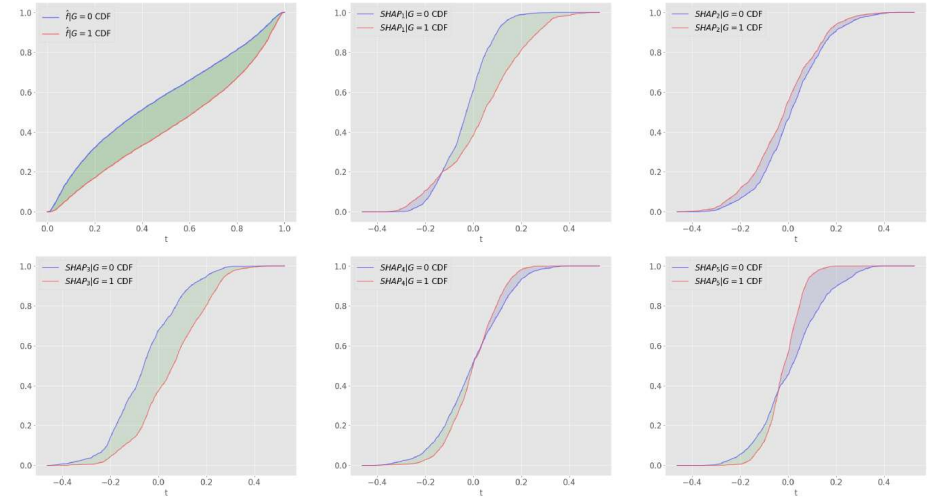
$$X_2 \sim \mathcal{N}(\mu - a_2(1 - G), 1)$$

$$X_3 \sim \mathcal{N}(\mu - a_3(1 - G), 1)$$

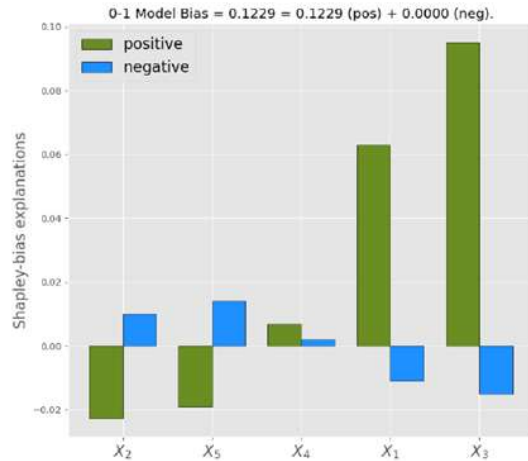
$$X_4 \sim \mathcal{N}(\mu - a_4(1 - G), 1 - 0.5G)$$

$$X_5 \sim \mathcal{N}(\mu - a_5(1 - G), 1 - 0.75G)$$

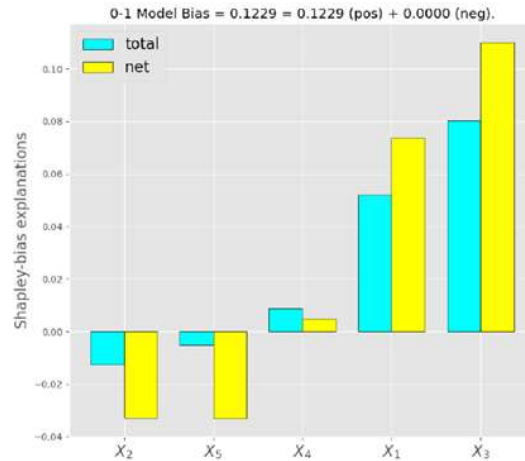
$$Y \sim \text{Bernoulli}(f(X)), f(X) = \sigma(\sum X_i - 24.5)$$



$$\varphi[v^{bias\pm}(\cdot, \varphi[v^{ME}])]$$



$$\varphi[v^{bias}(\cdot, \varphi[v^{ME}])]$$



## Stability of bias explanations

- **Conditional bias explanations** are consistent with the data; computational complexity might be infeasible under dependencies in  $X$
- **Marginal bias explanations are consistent** with the structure of the model  $f(x)$ , complexity  $O(2^n)$

### Lemma (stability [Miroshnikov et al 2021a])

The conditional and marginal Shapley-bias explanations have the following properties:

- $|\varphi_i^{bias\pm}(f|G, \varphi_S[v^{CE}]) - \varphi_i^{bias\pm}(f|g, \varphi_S[v^{CE}])| \leq C \|f - g\|_{L^2(P_X)}$
- $|\varphi_i^{bias\pm}(f|G, \varphi_S[v^{ME}]) - \varphi_i^{bias\pm}(f|g, \varphi_S[v^{ME}])| \leq C \|f - g\|_{L^2(\tilde{P}_X)}, \tilde{P}_X = \frac{1}{2^n} \sum_{S \subset N} P_{X_S} \otimes P_{X_{-S}}$

### Notes (Miroshnikov et al, 2021b, arXiv:2102.10878) :

- For marginal Shapley-bias explanations continuity in  $L^2(P_X)$  in general breaks down under dependencies in  $X$
- Marginal and conditional points of view can be unified via grouping and stability in  $L^2(P_X)$  is guaranteed
- Complexity can be reduced via quotient games and recursive approach

## Bias mitigation

Superposition [Miroshnikov et al, 2021c]

$$Bias_{W_1}(f|X, G) = \sum \bar{\beta}_i^{++} + \sum \bar{\beta}_i^{-+} - \sum \bar{\beta}_i^{+-} - \sum \bar{\beta}_i^{--} \geq 0$$

with  $\bar{\beta}_i^{\pm\pm} = \max(\varphi_i[v^{bias\pm}], 0)$ ,  $\bar{\beta}_i^{\pm-} = \max(-\varphi_i[v^{bias\pm}], 0)$

Special case (typical one)

Let  $f$  be positively-biased model, that is,  $Bias_{W_1}^+(f|X, G) > 0$ ,  $Bias_{W_1}^-(f|X, G) = 0$ . Then

$$Bias_{W_1}(f|X, G) = \sum \bar{\beta}_i^+ - \sum \bar{\beta}_i^- \geq 0$$

where  $\bar{\beta}_i^+ = \bar{\beta}_i^{++} + \bar{\beta}_i^{--}$ ,  $\bar{\beta}_i^- = \bar{\beta}_i^{-+} + \bar{\beta}_i^{+-}$ .

Note: This expression is the key to the bias mitigation procedure.



## Bias mitigation

The relationship  $Bias_{w_1}(f|X, G) = \sum \bar{\beta}_i^+ - \sum \bar{\beta}_i^- \geq 0$  is the key for bias mitigation via postprocessing:

1. Predictors with insignificant bias explanations are not relevant. This reduces dimensionality.
2. Adjusting the model so that  $\bar{\beta}_i^+ \downarrow$  and  $\bar{\beta}_i^- \uparrow$  should lead to model bias decrease.

**Question:** How to construct a postprocessed model  $\tilde{f}(X; f)$  that does not rely on  $(X, G)$ ?

## Bias mitigation

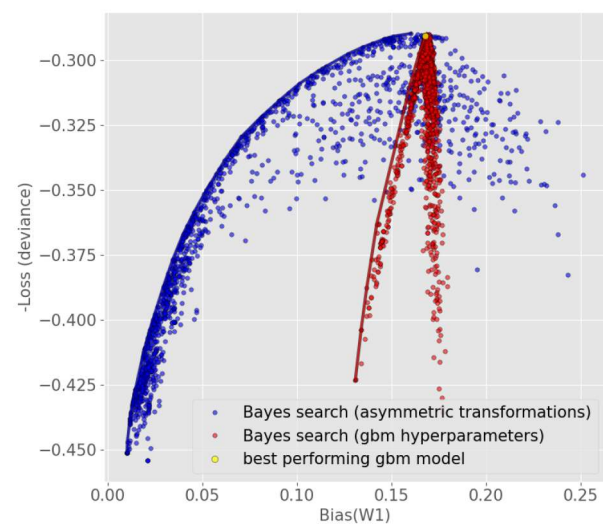
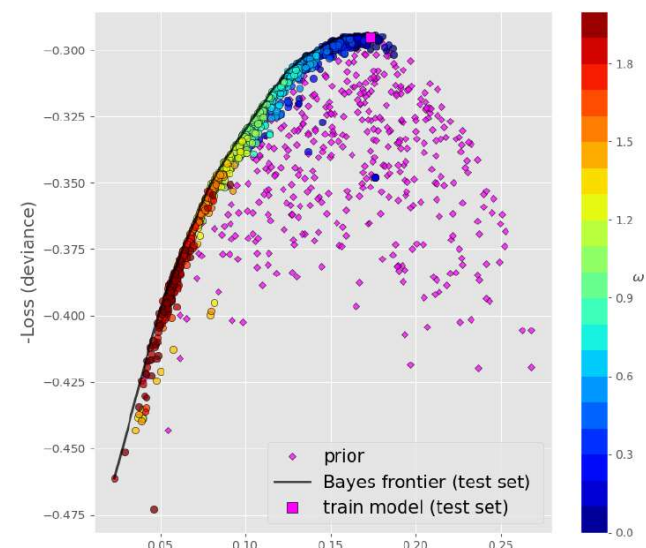
### Efficient frontier via rebalancing [Miroshnikov et al 2021c]

- $M = \{i_1, i_2, \dots, i_m\}$  most bias impactful predictors
- $\mathcal{F} = \{\tilde{f}: \tilde{f} = \mathcal{C}[f(T(X_M; \alpha), X_{-M})], \alpha \in A \subset \mathbb{R}^{mk}\}$
- $T(\cdot; \alpha)$  adjusts each predictor appropriately (scaling)
- $\mathcal{C}[\cdot]$  calibrates the distribution
- Efficient frontier is recovered by solving:

$$\alpha_*(\omega) = \operatorname{argmin}_{\tilde{f}} \{\mathbb{E}[L(Y, \tilde{f})] + \omega \cdot \operatorname{Bias}_{W_1}(\tilde{f}|X, G)\}$$

### Strategies for choosing $M$

1. Given  $m_*$ :  $N_{\pm} = \{i: m_*\text{-highest } \beta_i^{\pm}\}$ . Set  $M = N_+ \cup N_-$ .
2. Given  $m_*$ :  $M = \{i: m_*\text{-highest } \beta_i\}$ . Set  $M = N_+ \cup N_-$ .



# Bias mitigation

## Example

$$\mu = 5, a = \frac{1}{20}(10, -4, 16, 1, -3)$$

$$X_1 \sim \mathcal{N}(\mu - a_1(1 - G), 0.5 + G)$$

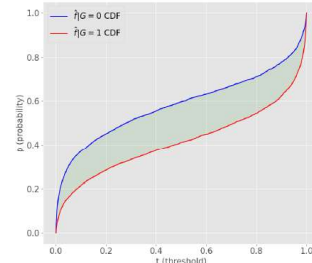
$$X_2 \sim \mathcal{N}(\mu - a_2(1 - G), 1)$$

$$X_3 \sim \mathcal{N}(\mu - a_3(1 - G), 1)$$

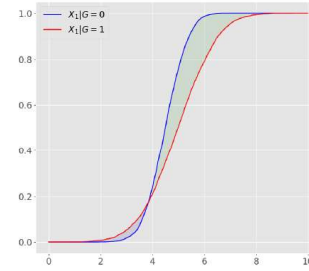
$$X_4 \sim \mathcal{N}(\mu - a_4(1 - G), 1 - 0.5G)$$

$$X_5 \sim \mathcal{N}(\mu - a_5(1 - G), 1 - 0.75G)$$

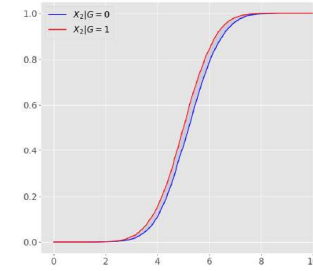
$$Y \sim \text{Bernoulli}(f(X)), f(X) = \sigma(2(\sum X_i - 24.5))$$



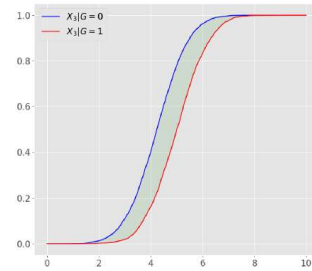
(a) Subpopulation distributions



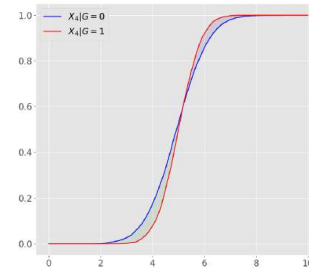
(b)  $X_1$  CDFs



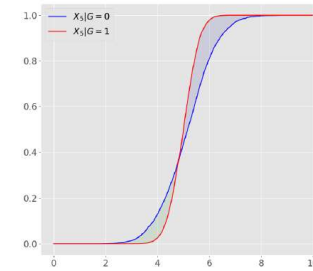
(c)  $X_2$  CDFs



(d)  $X_3$  CDFs



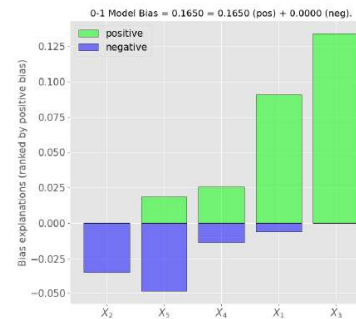
(e)  $X_4$  CDFs



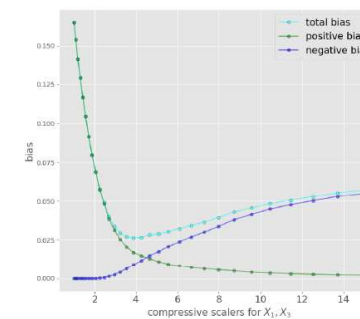
(f)  $X_5$  CDFs

## Effect of compression:

- Compressing  $X_1, X_3$  via a compressive map  $T(x_i; x_i^*)$
- Set  $\tilde{f} = f(T(X_1; x_1^*), X_2, T(X_3; x_3^*), X_4, X_5)$ ,  $x_i^* = \mathbb{E}[X_i]$



(g) Bias explanations



(h) Change in bias

## Acknowledgements

- Steve Dickerson (SVP, Chief Data Science Officer, Decision Management, Discover)
- Raghu Kulkarni (VP, Data Science, Discover)
- Melanie Wiwczaroski (Sr. Director, Enterprise Fair Banking, Discover)
- Melinda Milenkovich (VP & Assistant General Counsel, Discover)
- Kate Prochaska (Sr. Counsel & Director, Regulatory Policy, Discover)
- Markos Katsoulakis (Full Professor, University of Massachusetts Amherst)
- Robin Young (Full Professor, University of Massachusetts Amherst)
- Matthias Steinrücken (Assistant Professor, University of Chicago)

# References

- C. Dwork, M. Hardt, T. Pitassi, O. Reingold, and R.S. Zemel, Fairness through awareness. In Proc. ACM ITCS, 214-226, (2012).
- M. Feldman, S.A. Friedler, J. Moeller, C. Scheidegger, and S. Venkatasubramanian. Certifying and removing disparate impact. In Proc. 21st ACM SIGKDD, 259-268, (2015).
- J. H. Friedman, Greedy function approximation: a gradient boosting machine, *Annals of Statistics*, Vol. 29, No. 5, 1189-1232, (2001).
- P. Gordaliza, E. D. Barrio, G. Fabrice, J.-M. Loubes Obtaining Fairness using Optimal Transport Theory, Proceedings of the 36th International Conference on Machine Learning, PMLR 97:2357-2365, (2019).
- P. Hall, B. Cox, S. Dickerson, A. Ravi Kannan, R. Kulkarni, N. Schmidt, A United States Fair Lending Perspective on Machine Learning. *Front. Artif. Intell.* 4:695301. doi: 10.3389/frai.2021.695301 (2021).
- M. Hardt, E. Price, N. Srebro, Equality of opportunity in supervised learning. In *Advances in Neural Information Processing Systems*, 3315-3323, (2015).
- S.M. Lundberg and S.-I. Lee, A unified approach to interpreting model predictions, 31st Conference on Neural Information Processing Systems, (2017).
- A. Miroshnikov, K. Kotsiopoulos, R. Franks, A. Ravi Kannan, Wasserstein-based fairness interpretability framework for machine learning models, *arXiv preprint* (2021a), arXiv:2011.03156.
- A. Miroshnikov, K. Kotsiopoulos, A. Ravi Kannan, Mutual information-based group explainers with coalition structure for machine learning model explanations, *arXiv preprint* (2021b), arXiv:2102.10878.
- A. Miroshnikov, K. Kotsiopoulos, R. Franks, A. Ravi Kannan, Model-agnostic bias mitigation methods with regressor distribution control for Wasserstein-based fairness metrics, *arXiv preprint* (2021), arXiv:2111.11259
- A. Müller, Integral probability metrics and their generating classes of functions. *Advances in Applied Probability*, 29(2):429–443, (1997).
- V. Perrone, M. Donini, K. Kenthapadi, Cedric Archambeau Fair Bayesian optimization, ICML AutoML Workshop. 2020
- L. S. Shapley, A value for n-person games, *Annals of Mathematics Studies*, No. 28, 307-317 (1953).
- E. Strumbelj, I. Kononenko, Explaining prediction models and individual predictions with feature contributions. *Knowl. Inf. Syst.*, 41, 3, 647-665, (2014).
- Schmidt, N., Curtis, J., Siskin, B., and Stocks, C. Methods for Mitigation of Algorithmic Bias Discrimination, Proxy Discrimination, and Disparate Impact. U.S. Provisional Patent 63/153,692, (2021).
- B. Woodworth, S. Gunasekar, M. I. Ohanessian, and N. Srebro. Learning nondiscriminatory predictors. In Proc. of Conference on Learning Theory, p. 1920–1953, (2017).
- B. H. Zhang, B. Lemoine, M. Mitchell, Mitigating Unwanted Biases with Adversarial Learning. In Proc. of the 2018 AAAI/ACM Conference on AI, Ethics, and Society (pp. 335–340).